MATH 147 QUIZ 5 SOLUTIONS

1. Find the extreme value for $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $g(x, y, z) = x + y + z = 1$. Is this extreme value a minimum or a maximum? Justify your answer. (10 pts)

   We solve via the method of Lagrange multipliers. That is, since $g(x, y, z)$ describes a level surface, we set up the equation
   $$\nabla f(x) = \lambda \nabla g(x).$$
   This gives us the following equation
   $$(2x)\vec{i} + (2y)\vec{j} + (2z)\vec{k} = \lambda\vec{i} + \lambda\vec{j} + \lambda\vec{k}$$
   or the system of equations
   $$\begin{aligned} 2x &= \lambda \\ 2y &= \lambda \\ 2z &= \lambda. \end{aligned}$$
   This gives the solution $x = y = z$, and plugging this into the constraint gives $x = y = z = \frac{1}{3}$.
   This is a minimum. One can justify this by noting that f is unbounded above, and since $g(x, y, z) = 1$ describes a plane in $\mathbb{R}^3$, f will continue to increase as we travel along any of the axes. A similar method would be to check other points: $(1, 0, 0)$ lies on the plane $g = 1$, but $f(1, 0, 0) = 1 > f(1/3, 1/3, 1/3) = 1/3$. One can take this to the extreme and note that $(100, 0, -99)$ lies on the plane, but f has a massive value at this point. Geometrically, we note that f describes the square of the distance from the origin in $\mathbb{R}^3$, and so this minimum describes the point on the plane closest to the origin. Therefore, we can justify this is a minimum by noting that $g = 1$ can get as far from the origin as we want it. Finally, one may attempt the second derivative test, but this is not recommended.